Characteristics of Successful Opinion Leaders in a Bounded Confidence Model

Shuwei Chen[1,2], David H. Glass[2], and Mark McCartney[2]

1. School of Mathematics, Southwest Jiaotong University, Chengdu 610031, China
2. School of Computing and Mathematics, Ulster University
Shore Road, Newtownabbey, Co. Antrim, BT37 0QB, UK

swchen@swjtu.edu.cn, {dh.glass, m.mccartney}@ulster.ac.uk

ABSTRACT

This paper analyses the impact of competing opinion leaders on attracting followers in a social group based on a bounded confidence model in terms of four characteristics: reputation, stubbornness, appeal and extremeness. In the model, reputation differs among leaders and normal agents based on the weights assigned to them, stubbornness of leaders is reflected by their confidence towards normal agents, appeal of the leaders is represented by the confidence of followers towards them, and extremeness is captured by the opinion values of leaders. Simulations show that increasing reputation, stubbornness or extremeness make it more difficult for the group to achieve consensus, but increasing the appeal will make it easier. The results demonstrate that successful opinion leaders should generally be less stubborn, have greater appeal and be less extreme in order to attract more followers in a competing environment. Furthermore, the number of followers can be very sensitive to small changes in these characteristics. On the other hand, reputation has a more complicated impact: higher reputation helps the leader to attract more followers when the group bound of confidence is high, but can hinder the leader from attracting followers when the group bound of confidence is low.

Keywords: Opinion dynamics; Social networks; Bounded confidence model; Opinion leader; Characteristics

1. Introduction

Opinion dynamics in a group of interacting agents (individuals) has been studied for a long time from a wide range of perspectives, e.g., sociology, physics, politics, economics and philosophy [1-6]. In these models of opinion dynamics, a group of agents who hold beliefs and opinions about a given topic interact with each other to seek truth or reach consensus [7]. In many real situations, there are some agents, called *opinion leaders*, in a society who have more power on influencing the other agents' opinions because of their expertise or positions [8-10]. In some cases, there are usually two or more (groups of) opinion leaders holding competing opinions, e.g., political parties in elections, experts in scientific research, opinion leaders in science and religion [11, 12]. In a society with competing opinion leaders, questions such as, under what circumstances a) agents follow one of the opinion leaders but reject the other, b) opinion followers switch from following one leader to another, c) a sub-group of agents emerge that do not follow any opinion leader, immediately suggest themselves.

The original definition of opinion leaders in social opinion dynamics was provided by Katz and Lazarsfeld [13] as "the individuals who were likely to influence other persons in their immediate environment." Although intuitively clear, this definition does not provide a specific mechanism through which the opinion leaders influence the others. The well-known two-step flow model described a process of the information influence "flowing" from the media through opinion leaders to their respective followers [14], where the mechanics of the process itself remain unspecified. Based on the two-step flow model, using different opinion update rules, a significant number of opinion dynamics models have been built to analyse the function of opinion leaders in the context of party elections, marketing, and diffusion of innovations [8, 13, 15].

Despite the merits of these models, it is worth considering in more detail how to model the mechanism and measure the power of opinion leaders to influence their followers. In reality, the ways opinion leaders utilize their powers to affect the opinions of others are quite diverse due to different scenarios, their personalities and backgrounds [1, 16]. It is hardly possible to consider all of the real characteristics in one mathematical model, but key characteristics can be included. Kurmysheve and Juárez [17] suggested *reputation* as a key characteristic reflecting the power of opinion leaders, which is usually realized through higher weights assigned to leaders. Interestingly, however, Douven and Riegler [18] reported that applying higher reputation to some elite agents does not produce a discernible difference compared to the situation where all the agents are equally weighted. It was also suggested [17] that opinion leaders should maintain invariable opinions in the process of opinion dynamics, which is sometimes called *stubbornness* [19, 20]. More recently, Zhao and Kou [21] proposed an opinion leader-follower model which divides the whole group into two sub-groups, i.e., a leader sub-group and a follower sub-group, and the information / influence transition is based on the principle of a two-step flow model where the opinion update is based on a bounded confidence model, the Hegselmann-Krause (HK) model [22, 23]. Their results showed that the opinion followers are strongly influenced by the leaders if they have higher confidence levels and trust degrees in the leaders. The trust degree introduced in their model is defined by the weights of leaders, and so is actually a kind of reputation, while the higher confidence levels of followers towards opinion leaders can be thought of as the *appeal* of the opinion leaders.

Most of the existing models on opinion dynamics with opinion leaders discuss only one leader or a sub-group of opinion leaders who normally achieve consensus, while the phenomenon that two or more (groups of) opinion leaders hold competing opinions in a social group is very common in reality [11, 12], and therefore it is important to explore how competing opinion leaders can attract more followers. For potentially competing opinion leaders, their opinions usually go in polarized directions due to the competing environment [24]. Therefore, the opinion *extremeness* should also be considered as an important characteristic of opinion leaders, especially in a competing environment. Summing up, we conclude that the influential power of competing opinion leaders on their followers in a social group can be measured by four characteristics: reputation, stubbornness, appeal and extremeness. However, there are few studies that have analysed the impact of such characteristics on opinion dynamics in a social group with competing opinion leaders systematically, and therefore this will be the focus of the current paper.

As for opinion update mechanisms, there are generally two types of opinion dynamics models: continuous opinion dynamics and discrete opinion dynamics [5, 6]. In continuous opinion dynamics models, the opinion is usually modelled as a real variable in the interval [0, 1], and the agents interact with each other to update their opinions. The bounded confidence model is a representative continuous opinion dynamics model, where agents only interact with neighbours whose opinions are similar to theirs, and the similarity is decided by the bound of confidence or tolerance [25]. Among these models, the Deffuant-Weisbuch (DW) model and HK model have recently received considerable attention [26-33]. It has already been well established that these models have consensus thresholds for the bound of confidence, above which a consensus in the group is always achieved while the whole group may split into two or more non-interacting sub-groups with the same opinion in each of them when below the consensus thresholds [5]. Furthermore, several interesting modifications or extensions to these models have been introduced recently. Some of the modifications introduce heterogeneous bounded confidence such that the assumption that all individuals in a given society have the same level of confidence is no longer necessary [32]. The impact of external factors, e.g., mass media, on dynamics of opinions in real societies have also been analysed recently [34, 35]. In this paper we adopt the HK model as the basic opinion update mechanism for the group opinion dynamics with competing opinion leaders, and use it to investigate the impact of the four characteristics of opinion leaders identified above.

2. The HK model with opinion leaders

The HK model is based on a complete network consisting of n vertices, representing agents, i.e., all the agents are linked to each other. Each agent holds an opinion about a given topic. The opinion $x_i(t)$ that agent i has at time-step t is a real variable in the interval [0, 1], and it may change along a set of discrete time points according to the given update mechanism. We adopt the extended HK model proposed in [18] as the basic opinion update mechanism, which is

$$x_i(t+1) = \left(g(i,t)\right)^{-1} \sum_{j \in I(i,t)} r_j x_j(t) \ . \tag{1}$$

$I(i,t) = \{ j : \left| x_i(t) - x_j(t) \right| \le \varepsilon_i \}$ are epistemic neighbourhoods [23] of agent i at time t, and the parameters ε_i is the bound of confidence in opinion. Here the weight $r_j > 0$ represents the reputation that is assigned to agent j, and the function $g(i,t) = \sum_{j \in I(i,t)} r_j$ sums the reputations of agents in $I(i, t)$.

Since the original HK model does not consider opinion leaders, we need to incorporate their characteristics into the model in order to study their impact on the opinion dynamics. Suppose that there are two opinion leaders in a social group, who usually hold opposite opinions about a topic and are denoted as L_1 and L_2. The opinion leaders are assigned larger weights compared to normal agents to reflect their *reputation*. The reputations of opinion leaders L_i are denoted as R_i (i = 1, 2) and they are usually larger than r, the weight of normal agents which will always be 1. The *stubbornness* of opinion leaders is reflected by their small bounds of confidence, denoted as S_i (i=1, 2), so that they can influence the other agents but are not, or are less, influenced by them. We define the stubbornness of opinion leaders as $S_i = 1 - T_{L_i}$ (i=1, 2), where T_{L_i} denotes the bound of confidence of leader L_i. On the other hand, the normal agents should have larger bounds of confidence towards the leaders than that towards their peers (denoted as T), which are denoted as A_i (i = 1, 2), reflecting the *appeal* of opinion leaders to the normal agents. Finally, the *extremeness* of opinion leaders is reflected by how close they are to the extreme values, 0 and 1, of the opinion interval, and it is defined as $E_i = 2 \times \left| 0.5 - x_{L_i} \right|$ (i=1, 2), where x_{L_i} is the opinion value of leader L_i. Note that extremeness itself cannot make an agent an opinion leader, and so we do not investigate its impact by itself but the combined effect with the other characteristics.

Since we are mainly interested in how the opinion leaders compete to attract followers, we adopt the average number of stationary sub-groups of normal agents (i.e., we exclude groups consisting of only opinion leaders), and the average number of followers of the opinion leaders as the quantities to study the impact of these four characteristics. There are different ways to define what it means for an agent to be a follower of a given opinion leader. One might define followers as the agents with the same opinion as the leader in the post transient regime. This definition seems too strict, however, because some agents may have a similar, but not exactly the same, opinion as the leader and their opinions may become identical to that of the leader if some conditions changed slightly. Therefore, a more reasonable approach is to define opinion followers as the agents whose opinions are within a certain range of the opinion of some opinion leader. Concretely, if one and only one opinion leader is within the epistemic range, as defined in equation (1), of one agent when stability is achieved, then this agent is called an *opinion follower* of that leader. If the two leaders are both within the epistemic range of an agent, then the agent is being influenced by both leaders and cannot be called a follower of either leader.

3. Simulation results and discussion

We consider a society with two competing opinion leaders and investigate the impact of the four characteristics one by one. In addition, these two opinion leaders do not interact with each other due to the nature of competition, i.e., they are completely stubborn towards each other. Before going into the detail of the simulations, we briefly discuss the real world interpretations of some parameter settings.

Number of agents – It is possible for a real world network to have a very low number of agents or a large number of agents. In the performed simulations, the number of agents is chosen as 200, and two of the agents are designated opinion leaders based on the specifically assigned opinions as described in the following paragraph. We have implemented the simulations on the network with up to 1000 agents and found that there was no significant difference in the results compared to the network with 200 agents.

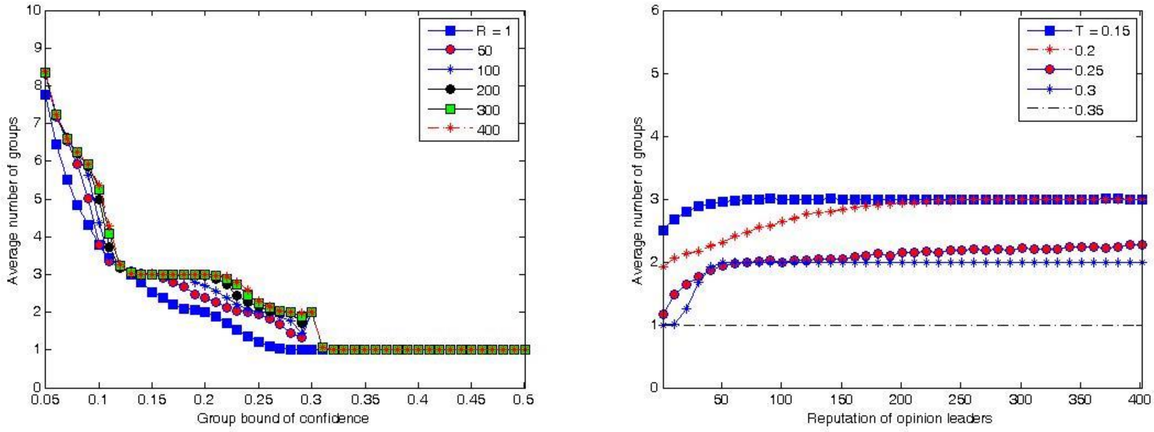
Initial opinion values are generated randomly on the interval [0, 1] (uniformly distributed) for each agent, while that of the opinion leaders are set to some specific values, i.e., a smaller value of 0.2 for leader L_1 and a symmetric larger value of 0.8 for leader L_2 when exploring the impact of reputation, stubbornness and appeal. The symmetry in leaders' opinions helps to see clearly the impact of these three characteristics. We also vary the opinion of leader L_1 from 0 to 0.5 and leader L_2 from 1 to 0.5 when investigating the impact of extremeness. It is worth mentioning that when testing the impact of the four characteristics of opinion leaders, we use the same initial opinions and apply different values to the parameters corresponding to these characteristics.

Repeated runs – Given the fact that the initial opinions are generated randomly, simulations might show variant results even with the same parameter settings. For this reason, we have implemented 200 and 500 runs with all the other conditions being the same, and found that 200 runs were sufficient to obtain reasonable results.

3.1. Impact of reputation on opinion dynamics

As stated previously, it has been reported that reputation does not produce a discernible difference in agent dynamics by applying higher weights to some elite agents compared with the situation where all the agents are equally weighted [18]. We therefore let the reputation of opinion leaders R_i vary in a wider range, from 1 to 400 in steps of 10, to see its impact, while keeping the values of other characteristics the same as that of the normal agents except for the starting initial opinion values. That is we set $T_{L_i} = A_i = T$, $E_i = 0.6$, where T is the bounds of confidence of normal agents.

First we investigate the average number of stationary sub-groups of normal agents with the reputations of both opinion leaders varying simultaneously from 1 to 400 in steps of 10 while the group bound of confidence T takes values from 0.05 to 0.5 in steps of 0.01. First of all, as has been studied in the literature, Fig. 1 (a) shows that increasing the group bound of confidence will make the agents more likely to achieve consensus. As for the impact of reputation, Fig. 1 (a) demonstrates that increasing reputation will slightly increase the number of sub-groups, i.e., make it harder for the agents to achieve consensus, when the group bound of confidence $T \in (0.15, 0.3)$. For most values of T, this effect is significant only when the reputation is smaller than 50, as is further shown in Fig. 1 (b), while there is little effect in terms of the number of sub-groups by further increasing the reputation, which confirms the findings in [18]. However, even small changes might be significant in certain respects. For example, the figures also show the change of consensus threshold from 0.25 when there is no opinion leader (R_i=1) to around 0.32 when the leaders have a higher reputation R_i=50 (i = 1, 2). The reason for this is that when is the opinion leaders have a sufficiently high reputation, e.g., 50, their neighbours converge to them more quickly because of the higher reputation and the others that fall outside their epistemic range will have less chance to be influenced by them either directly or indirectly via their neighbours. However, when the group bound of confidence is larger than 0.32 as shown in the figures, there will be an overlap between the epistemic ranges of the two leaders, which makes the leaders affect each other via their common neighbours and makes the group achieve consensus.



(a) (b)

Fig. 1. Average number of sub-groups with respect to (a) group bound of confidence *T* with different reputation values, and (b) reputation R_i with different bounds of confidence, where S_i =1-*T*, A_i = *T*, E_i = 0.6. Averages taken over 200 runs.

In order to see the impact of reputation for the two opinion leaders on attracting followers, we then explore the average number of followers of the two opinion leaders as shown in Fig. 2 by fixing the reputation of opinion leader L_2 to, R_2 =50, and varying the reputation of leader L_1 from 1 to 400 in steps of 5, while keeping the values of other characteristics the same as in the above simulations. We have also investigated the case where R_2 =100, and the simulation results show a similar pattern.

When the group bound of confidence *T* is 0.2 as shown in Fig. 2 (a), increasing the reputation of leader L_1 actually *decreases* the number of his followers while increasing the number of followers of leader L_2. The two leaders are always attracting the same number of followers when *T* = 0.3 as shown in Fig. 2 (b), but increasing the reputation of leader L_1 increases the number of followers of both leaders dramatically until $R_1 = R_2 = 50$ and thereafter all the agents are divided equally into two groups of followers of the two leaders. When *T* = 0.35 as shown in Fig. 2 (c), neither leader can attract any followers when $R_1 < 100$, and when R_1 increases from 100 to 230, the number of followers of both leaders increases quickly, but leader L_1, who has higher reputation, is able to gain more followers now. The state that neither leader can attract any follower lasts longer when *T* = 0.4 as shown in Fig. 2 (d) until R_1 = 300 and thereafter both leaders start to attract a few followers with increasing reputation of leader L_1.

The counterintuitive result that higher reputation results in fewer followers in some cases can be explained by reasoning along the lines stated earlier. The opinion leader with higher reputation is able to persuade their epistemic neighbours converge to them more quickly and the others that fall outside their epistemic range will have less chance to be influenced by them either directly or indirectly. On the other hand, the leader with lower reputation can work with their epistemic neighbours and persuade the agents that were outside of their epistemic range to follow them via their common neighbours although the process may take a bit longer. As a result, when the group bound of confidence T takes a smaller value as shown in Fig. 2 (a), the opinion leader with higher reputation can only influence the agents in a narrow range and gain less followers. However, when T becomes larger as shown in Fig. 2 (b), the leader with higher reputation is able to influence a wider range of agents, which overcomes the previous negative impact of reputation gradually. When T is larger still as shown in Fig. 2 (c) and (d), all of the normal agents are able to be influenced by both leaders and will normally achieve consensus, and only when one leader has a much higher reputation than the other can this enable that leader to attract more followers. The consensus can be achieved because of the emergence of an intermediate group that brings the two opinion leaders and their followers together. However, for higher reputations this intermediate group can instead be incorporated into the followers of the opinion leader with the highest reputation.

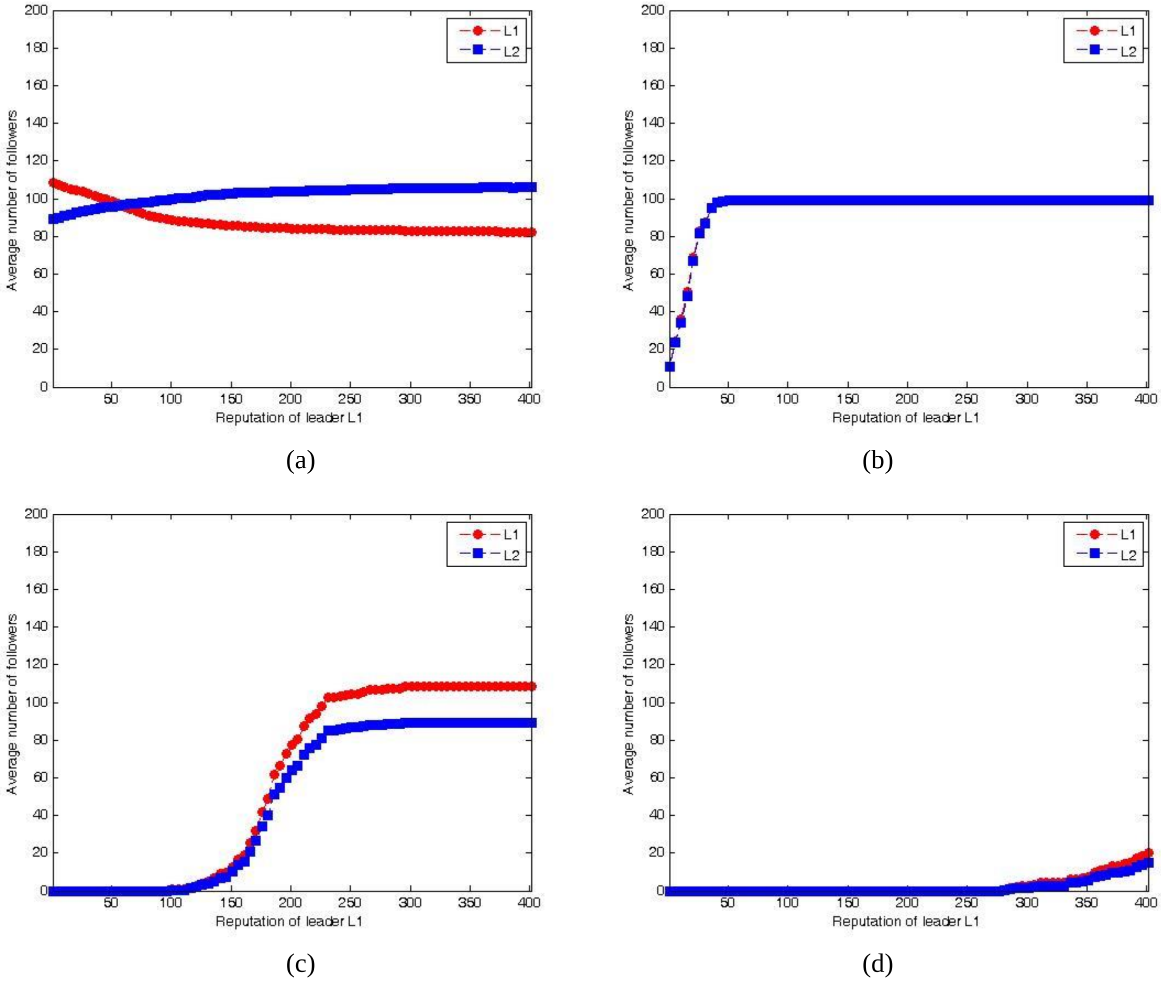
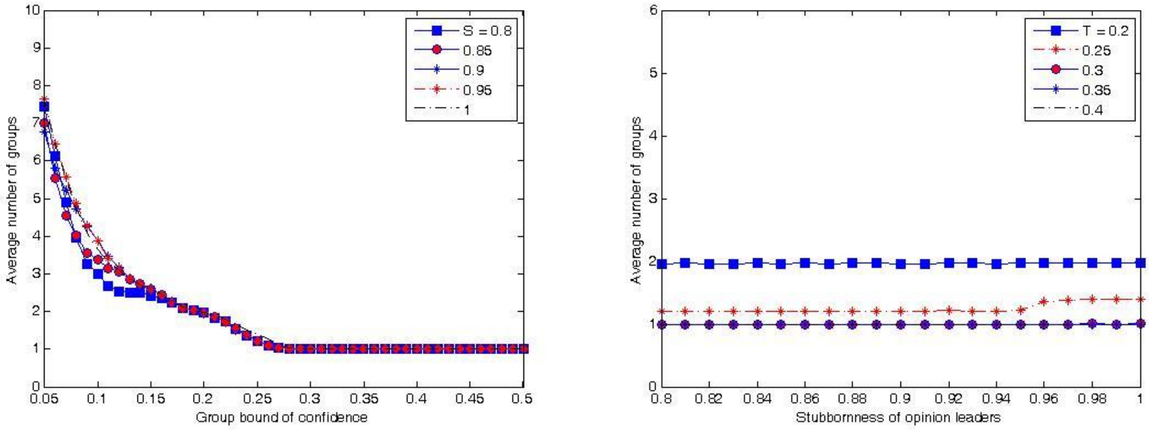


Fig. 2. Average number of followers of opinion leaders L_1 and L_2 with respect to reputation of leader L_1, where R_2 =50, S_i = 1-T, A_i = T, E_i = 0.6 and group bound of confidence T is (a) 0.2, (b) 0.3, (c) 0.35 and (d) 0.4. Averages taken over 200 runs.

3.2. *Impact of stubbornness on opinion dynamics*

To study the impact of the stubbornness of opinion leaders on the group opinion dynamics, taking a similar strategy as when investigating the impact of reputation, we first let $A_i = T$, $R_i = 1$, and S_i vary simultaneously from 0.8 to 1 in steps of 0.01 (the corresponding bounds of confidence for the opinion leaders T_{L_i} change from 0.2 to 0) while the group bound of confidence T takes values from 0.05 to 0.5 in steps of 0.01, and explore the average number of sub-groups of the normal agents. The simulation results are shown in Fig. 3.

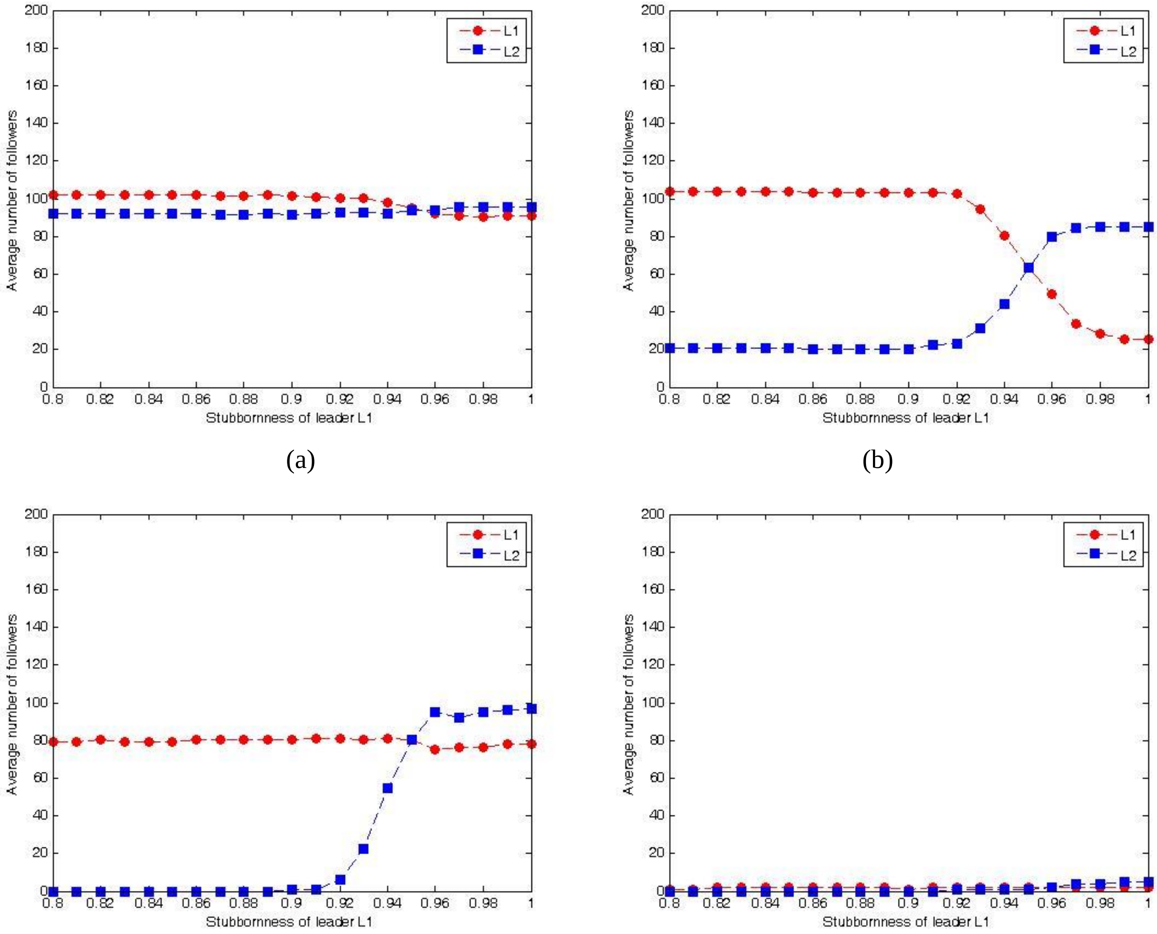


(a) (b)

Fig. 3. Average number of groups with respect to (a) group bound of confidence T with different stubbornness values, and (b) stubbornness S_i with different bounds of confidence, where R_i =1, $A_i = T$, $E_i = 0.6$. Averages taken over 200 runs.

Overall, the simulation results in Fig. 3 show that stubbornness has little effect on the group opinion dynamics in terms of average number of sub-groups. Increasing stubbornness of the leaders results in a slight increase in the number of sub-groups only when the group bound of confidence T is around 0.1 as shown in Fig. 3 (a). Fig. 3 (b) also illustrates that the average number of groups is almost independent of stubbornness and only when the stubbornness increases from 0.95 to 0.96 at T = 0.25, does the average number of sub-groups become slightly larger. For example, where the same number of sub-groups are formed under different stubbornness degrees, it takes a longer time (about 60 to 100 times longer) for the agents to reach stationary state if the leaders are completely stubborn than if the stubbornness is 0.9.

In order to see the impact of stubbornness of opinion leaders on attracting followers, we similarly explore the average number of followers of the two opinion leaders by letting opinion leader L_2 be $S_2 = 0.95$ and varying the stubbornness degree of leader L_1 from 0.8 to 1 in steps of 0.01. The simulation results are shown in Fig. 4.



(a) (b) (c) (d)

Fig. 4. Average number of followers of opinion leaders L_1 and L_2 with respect to stubbornness of leader L_1, where R_i =1, S_2=0.95, $A_i = T$, E_i = 0.6 and group bound of confidence T is (a) 0.2, (b) 0.25, (c) 0.3 and (d) 0.35. Averages taken over 200 runs.

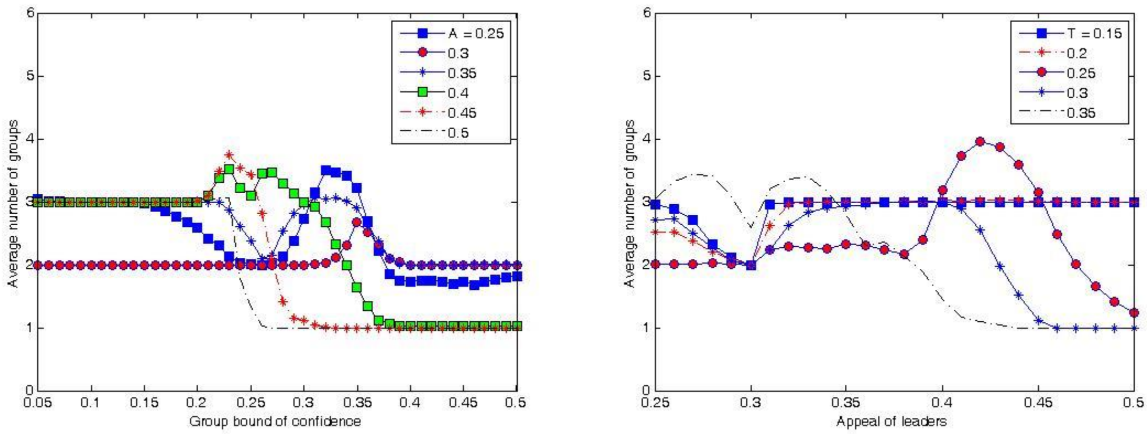
The figures when the group bound of confidence is smaller than 0.2 are not shown here since the normal agents would be more stubborn than the leaders at some points. It can be seen from Fig. 4 that the less stubborn leader is usually able to attract more followers, and the impact is more significant when the group bound of confidence T = 0.25 and 0.3 as shown in Fig. 4 (b) and (c). When T = 0.25, the number of followers of leader L_1 decreases dramatically from 100 to around 20 while that of leader L_2 increases from 20 to around 80 when the stubbornness of leader L_1 increases from 0.92 to 0.98. When T = 0.3, the number of followers of leader L_2 increases dramatically from 0 to 100 while that of leader L_1 keeps around 80 when the stubbornness of leader L_1 increases from 0.92 to 0.96. The transition between having more followers and fewer followers occurs at S_1 = 0.95 since that is the point where $S_1 = S_2$. The reason that lower stubbornness leads to more followers in general is that a less stubborn leader is more likely to learn the opinions of the normal agents by changing his opinion to be closer to theirs, and when the group bound of confidence is at certain range, e.g., 0.25 or 0.3, which is in the region of the consensus threshold when the leaders are equally stubborn as shown in Fig. 3 (a), the less stubborn leader is able to affect a sufficient range of agents and attract them as followers. When the group bound of confidence becomes larger than 0.35 as in Fig. 4 (d), both the leaders cannot attract followers anymore because all the normal agents are influenced by both the leaders (directly or indirectly) and this normally results in a consensus at the average opinion of the leaders. We have also carried out the simulations with $A_i = T$ and $R_i = 50$ and found that stubbornness has no discernible impact on attracting followers for the two leaders. The reason is that higher reputation of leaders makes their epistemic neighbours converge to them more quickly as stated in the previous section and the effect of stubbornness becomes irrelevant as a consequence since the opinions of the leaders do not change much even for low values of stubbornness.

We let the leaders to be completely stubborn, i.e., $S_i = 1$ when investigating the impact of appeal and extremeness in the following sections, because it helps to see their impact more clearly by fixing the leaders' opinions. For consistency, we also fix the reputation of opinion leaders to be $R_i = 50$, as it has been shown in Fig. 1 that much higher reputations makes little difference.

3.3. Impact of appeal on opinion dynamics

This section shows the impact of the appeal of opinion leaders on group opinion dynamics and attracting followers. With a similar strategy as before, we first investigate the average number of sub-groups of the normal agents by letting the appeal of both the leaders vary simultaneously from 0.25 to 0.5 in steps of 0.01 and the group bound of confidence T from 0.05 to 0.5 in steps of 0.01, while $R_i = 50$ and $S_i = 1$ ($i = 1, 2$). The simulation results are shown in Fig. 5.

The results in Fig. 5 show that increasing the appeal of both the leaders makes it easier for the normal agents to achieve consensus (i.e. it lowers the consensus threshold), which confirms the findings in [21], because they are more likely to be influenced by both leaders which makes them achieve consensus roughly at the average opinion of the leaders. There is a common pattern for most of the appeal values in Fig. 5 (a): with increasing group bound of confidence, the average number of sub-groups keeps stable at two or three at first, and rises to a peak with less than one additional sub-group on average, and then falls back to one normally (i.e., the agents achieve consensus). When the appeal of leaders is 0.3 as shown in Fig. 5 (a), the normal agents will usually be divided into two groups following the corresponding leaders. When the appeal is larger than 0.3 and the group confidence is low, the normal agents whose original opinions are around 0.5 are affected by both the leaders and will normally achieve consensus in the middle, but those who are far from the middle can only be affected by one of the leaders and become their followers resulting in three sub-groups in total. It is also shown in Fig. 5 (b) that, for several cases, the number of sub-groups changes from about three to two, when the appeal increase from 0.25 to 0.3, but back to three again with larger appeal. When the group bound of confidence *T* is 0.25, the number of sub-groups stays at 2 until the appeal is around 0.38 at which point it rises to the peak of four and gradually back to one again. This is because 0.3 is the distance between the middle of opinion interval and the opinions of the two leaders. When the leaders' appeal is between 0.25 and 0.3, the leaders can only attract the nearby normal agents quickly to become their followers because of their higher reputation, while the agents around the middle will not be affected by them.



(a) (b)

Fig. 5. Average number of groups with respect to (a) group bound of confidence *T* with different appeal values, and (b) appeal A_i with different group bounds of confidence where R_i = 50, S_i = 1, E_i = 0.6. Averages taken over 200 runs.

We can then investigate the impact of appeal on attracting followers for the leaders via the average number of followers of the opinion leaders, by fixing the appeal of opinion leader L_2 to be A_2 = 0.4, and varying that of leader L_1 from 0.2 to 0.5 in steps of 0.05, while R_i = 50 and S_i = 1 (i = 1, 2). The simulation results are shown in Fig. 6.

The results in Fig. 6 overall demonstrate that appeal plays a straightforward role for opinion leaders, i.e., increasing appeal normally helps the leader to attract more followers or lower the number of followers of the opposite leader. The reason is that the leader with higher appeal is normally able to affect a wider range of agents. However, if the group bound of confidence is large enough as shown in Fig. 6 (b) and (c), increasing the appeal will make more normal agents be influenced by both leaders and achieve consensus by themselves, and therefore reduce the number of followers of both leaders, especially in the sensitive appeal interval (0.35, 0.45) where the number of followers of leader L_2 decreases dramatically. When the group bound of confidence becomes larger still as shown in Fig. 6 (d), all the agents will be affected by both leaders and achieve consensus at the average opinion when the appeal of leader L_1 is greater than 0.4, and so neither leader can attract followers anymore. However, when the appeal of L_1 is smaller than 0.35, leader L_2 attracts all the normal agents as followers. This dramatic change in support for leader L_2 as the appeal of leader changes L_1 from 0.35 to 0.4 can be explained as follows. When the appeal of L_1 is smaller than that of leader L_2, A_2 = 0.4, and the group bound of confidence T = 0.4, the normal agents with opinions smaller than 0.4 will be pulled to the middle of the opinion interval at first due to the effect of both leaders (indirectly in the case of L_2) and other normal agents, but then further pulled up by L_2 and other normal agents, and become followers of L_2 subsequently.

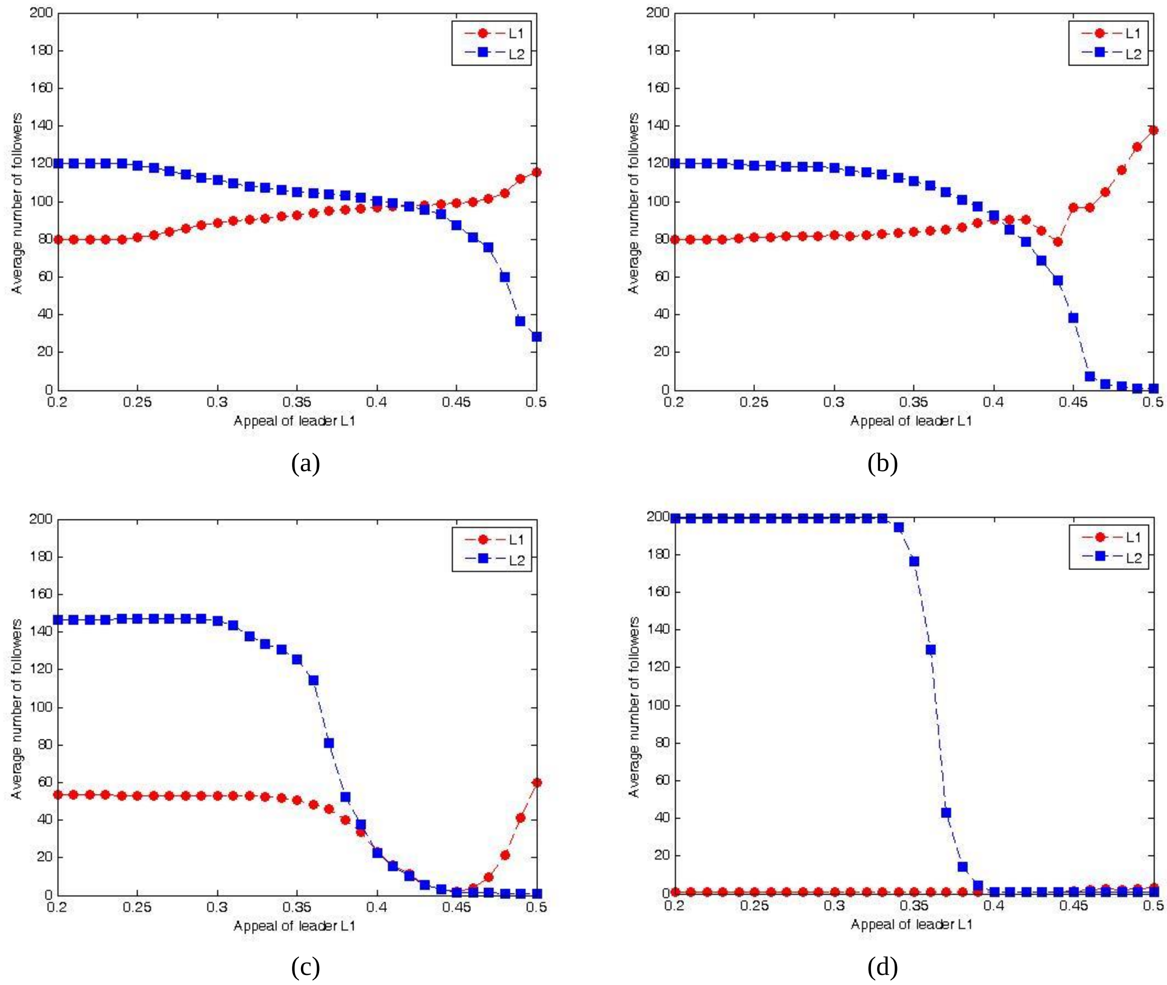
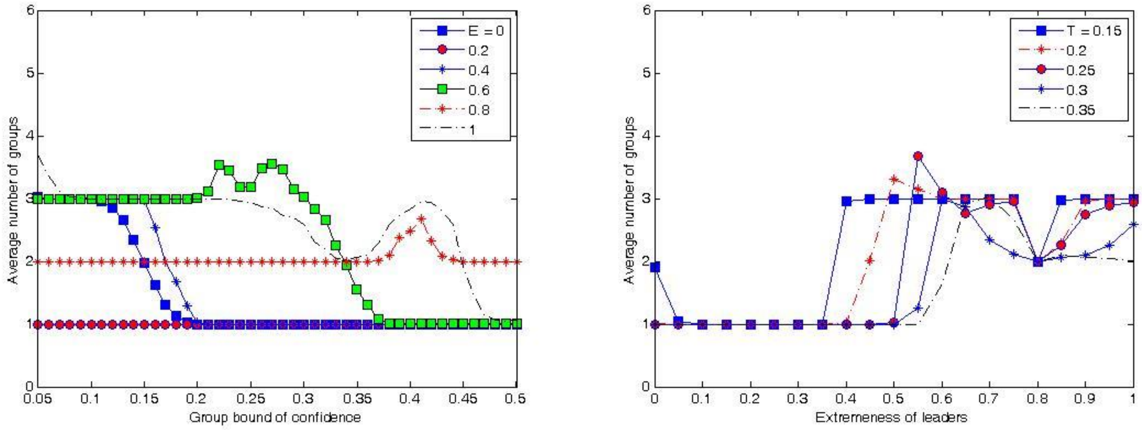


Fig. 6. Average number of followers of opinion leaders L_1 and L_2 with respect to appeal of leader L_1, where R_i =50, S_i=1, A_2 = 0.4, E_i = 0.6 and group bound of confidence T is (a) 0.25, (b) 0.3, (c) 0.35, and (d) 0.4. Averages taken over 200 runs.

3.4. Impact of extremeness on opinion dynamics

Similarly, we first investigate the average number of sub-groups of the normal agents by letting the opinion of leader L_2 change from 0.5 to 1 and that of leader L_1 vary symmetrically from 0.5 to 0 in steps of (±)0.025, which means that the extremeness of both leaders E_i vary simultaneously from 0 to

1 in steps of 0.05, and the group bound of confidence T vary from 0.05 to 0.5 in steps of 0.01, while R_i =50, S_i =1, and A_i = 0.4 (i = 1, 2).



(a) (b)

Fig. 7. Average number of groups with respect to (a) group bound of confidence T with different extremeness values, and (b) extremeness E_i with different group bounds of confidence where R_i = 50, S_i = 1, A_i = 0.4. Averages taken over 200 runs.

Overall the results in Fig. 7 show that in most cases it is easier for normal agents to achieve consensus when the leaders are less extreme. The reason is that, when the two leaders are moving simultaneously from the completely extreme opinions, 0 and 1, to the middle of the opinion interval, more normal agents will be affected by both of them and are more likely to form consensus. Apart from the overall effect, some critical extremeness values are worth discussing due to their interesting behaviour. When the extremeness is 0.2 as in Fig. 7 (a), i.e., opinion of leader L_1 is 0.4 and that of leader L_2 is 0.6, all the normal agents can be influenced by both leaders sooner or later given the fact that the appeal of the leaders is 0.4, and so the normal agents can always achieve consensus. When the extremeness is 0.8, i.e., opinion of leader L_1 is 0.1 and that of leader L_2 is 0.9, the normal agents will usually be divided into two sub-groups of followers of the leaders.

In order to investigate the impact of extremeness on attracting followers, we explore the average number of followers of the two opinion leaders by varying the extremeness of leader L_1 from 0 to 1 in steps of 0.05, and fix that of leader L_2 to be 0.6, i.e., E_2 = 0.6. The following figures show the simulation results for the average numbers of followers of opinion leaders L_1 and L_2 with respect to different group confidence levels where R_i =50, S_i =1, and A_i = 0.4 (i = 1, 2).

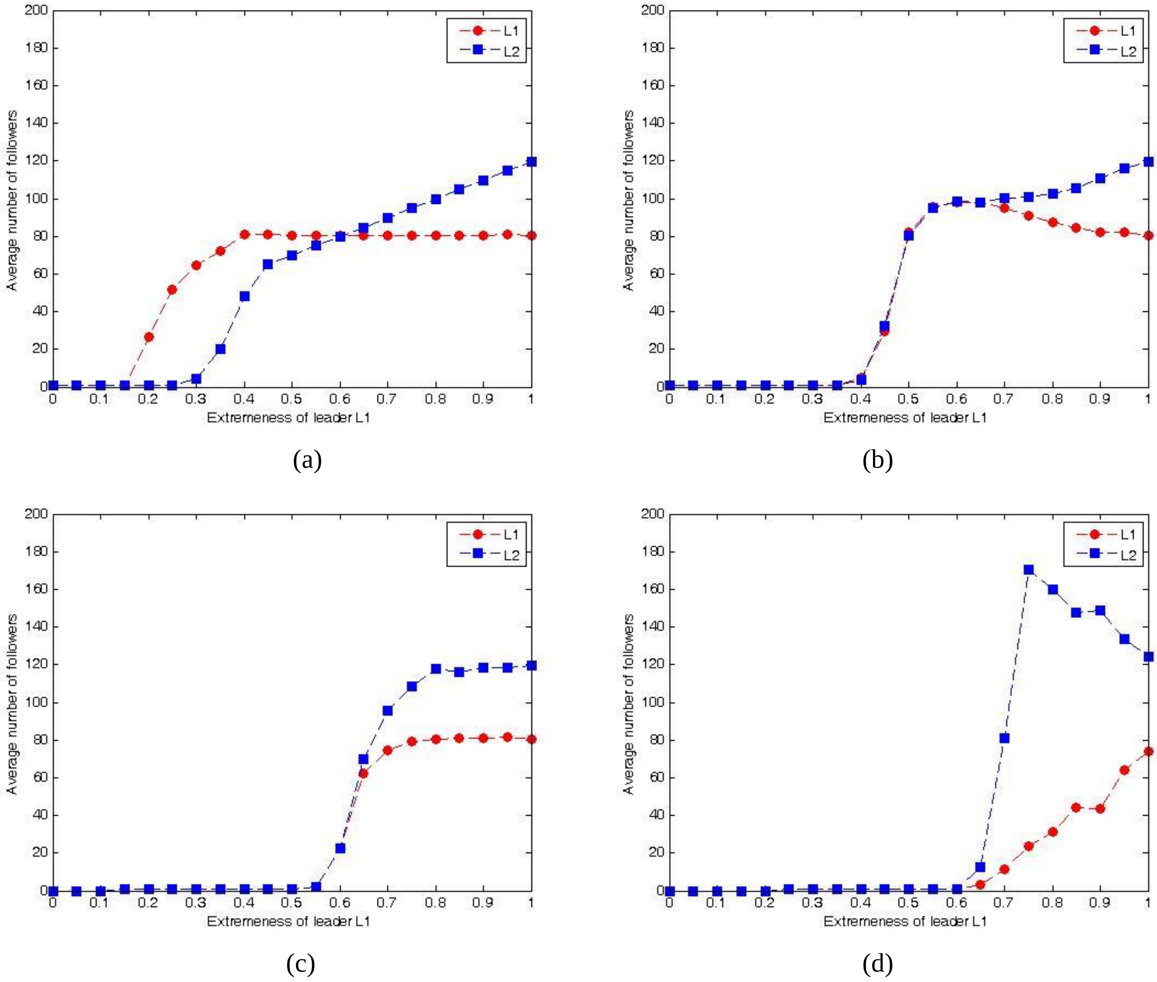


Fig. 8. Average number of followers of opinion leaders L_1 and L_2 with respect to extremeness of opinion leader L_1, where R_i =50, S_i =1, A_i =0.4, E_2 =0.6 and group bound of confidence T being (a) 0.2, (b) 0.25, (c) 0.35 and (d) 0.4. Averages taken over 200 runs.

Clearly, the extremeness of leader L_1 being 0.6 is a point where the numbers of followers of the two leaders are always the same since they have the same value for all four characteristics. When the extremeness of the two leaders differs, we can conclude from the above figures that the less extreme opinion leader is generally more capable of attracting followers. It is also shown that there are some critical extremeness values so that neither leader can attract any follower if E_1 is smaller than the critical value, but there will be dramatic increase in the number of followers of both leaders if E_1 becomes larger than the critical value, e.g., E_1 = 0.4 when T = 0.25 as in Fig. 8 (b), and E_1 = 0.6 when T = 0.4 as in Fig. 8 (d). This is because more agents will be able to be affected by both leaders indirectly with a larger group bound of confidence provided the opinion leaders are not too extreme and then they will normally achieve consensus by themselves and are not treated as the followers of any leader according to the definition. Only when the leaders are more extreme as stated above, do they depart from this state and the normal agents are broken from consensus into two sub-groups of followers.

4. Conclusions

This paper has introduced opinion leaders holding competing opinions, and analysed the impact of four characteristics of the opinion leaders – reputation, stubbornness, appeal and extremeness – on the group opinion dynamics and the ability of the opinion leaders to attract followers. It has been shown that increasing the reputation of opinion leaders may make it more difficult for the normal agents to achieve consensus, and the leader with higher reputation will make the neighbours converge to his opinion more quickly, and so attract more followers, when the group bound of confidence is larger, but fewer followers when the group bound of confidence is smaller. Stubbornness has no discernible effect on the number of sub-groups, but less stubborn leaders are usually able to attract more followers. On the other hand, increasing appeal of opinion leaders makes it easier for the normal agents to achieve consensus. It normally makes more agents begin to follow the opinion leader more quickly if a higher appeal is applied to that leader. The simulation results also demonstrate that increasing extremeness makes it more difficult for the agents to achieve consensus, and a less extreme opinion leader is usually able to attract more followers. While the general dependence on stubbornness, appeal and extremeness might be expected, the number of followers attracted by the opinion leaders can be very sensitive to these characteristics of the respective leaders. For example, in some cases a leader who is only slightly less (more) stubborn than the competing leader can attract a lot more (less) followers.

Note that since there are four characteristics for the two competing opinion leaders under discussion, it is impossible to explore the complete parameter space in one paper. As a result, we usually fix the values of other characteristics when investigating the impact of one of the characteristics. On the other hand, opinion leaders could of course try to strengthen one aspect when they are weak in the other aspect, and there will be compensation among different characteristics on their impact on group opinion dynamics. For instance, the opinion leader who does not have higher reputation but does have stronger appeal compared to the other leader might also be able to attract more followers. We leave this kind of compensation among different characteristics for future work. Note also that the values of the four characteristics are invariant during the opinion update process in this paper, but in reality the opinion leaders might change their strategies with respect to the four characteristics at certain stages in order to attract more followers. We are exploring some models with dynamic parameter settings to investigate this case.

It is also worth mentioning that the impact of external information, e.g., mass media [35], is not explicitly considered in our model, because it can be treated as a completely stubborn opinion leader.

Higher connectivity is also considered as a characteristic of opinion leaders in some papers [25], but as suggested in [17], it might not be an intrinsic feature of a leader but a topological property of the network reflecting a social position of the hub. We will discuss the impact of connectivity or different network structures relating to opinion leaders in future work. Although it seems out of control of opinion leaders, the group bound of confidence is definitely an important characteristic affecting group opinion dynamics, as it determines to what extent normal agents are 'open-minded' towards their peers. It has been shown in this paper that there is usually a tradeoff between the group bound of confidence and the four characteristics, and it is worth further exploring this relationship in future work.

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